POZNAN UNIVERSITY OF TECHNOLOGY

EUROPEAN CREDIT TRANSFER AND ACCUMULATION SYSTEM (ECTS)

COURSE DESCRIPTION CARD - SYLLABUS

Course name

Elements of Numerical Analysis [S1Inf1>EAN]

Course

Field of study	Year/Semester
Computing	1/2
Area of study (specialization)	**Profile of study**
–	general academic
Level of study	**Course offered in**
first-cycle	Polish
Form of study	**Requirements**
full-time	elective

Number of hours

Lecture	Laboratory classes	Other (e.g. online)
24	0	0
Tutorials	**Projects/seminars**	
24	0	

Number of credit points

5,00

Coordinators

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Lecturers

Prerequisites

A student beginning this subject should have a basic knowledge of algebra and mathematical analysis, be able to solve analytically tasks in these subjects, and have the ability to obtain information from the indicated sources. He should also understand the need to expand his competence. In terms of social competence, the student must represent such attitudes as honesty, responsibility, perseverance, cognitive curiosity, creativity and respect for other people.

Course objective

To learn the theoretical and practical problems of numerical methods, in particular: - understanding the implementation of floating point arithmetic on computers and the errors associated with it, - learning basic numerical methods for interpolation, approximation, solving systems of linear equations, and solving equations and systems of nonlinear equations, - acquiring the ability to estimate errors of calculations on computers, - develop the ability to solve various numerical tasks on computers, - perfecting the algorithmic approach to solving tasks.

Course-related learning outcomes

Knowledge

1. has an extended and deepened knowledge of mathematics (numerical analysis) useful for formulating and solving complex computer tasks [K1st_W1].
2. has a structured and theoretically supported general knowledge of the key issues of numerical analysis [K1st_W4].
3. knows the basic methods and tools used in the process of solving computer tasks (mainly of inynier nature) in the field of numerical analysis [K1st_W7].
Skills
1. is able to properly plan experiments in the field of numerical analysis, interpret the obtained results and correctly draw conclusions from them [K1st_U3].
2. is able, when formulating and solving computer tasks, to apply appropriate algorithms from the field of numerical methods [K1st_U4].
3. has the ability to formulate algorithms and their implementation from the field of numerical analysis [K1st_U11].
Social competences
He is aware of the importance of knowledge of numerical methods in solving engineering problems [K1st_K2].

Methods for verifying learning outcomes and assessment criteria

Learning outcomes presented above are verified as follows:

Formative assessment:
(a) in terms of lectures - on the basis of answers to questions on the material discussed in previous lectures,
b) in terms of laboratories - on the basis of the evaluation of the current progress of the tasks.
Summative assessment
Verification of the established learning outcomes is carried out by:
- continuous evaluation at each class (solving tasks on the blackboard), bonus of the increment of skills in the use of the learned methods,
- evaluation of knowledge and skills related to solving tasks through a test during the last laboratory classes,
- evaluation and defense by the student of a report on the implementation of a project involving the writing of a program (in any programming language) solving a selected computational problem with a specified method in ordinary and interval floating point arithmetic, and the execution of documentation for this program,
- evaluation of knowledge and skills demonstrated in a written exam of theoretical and problem-based (the exam includes a theoretical description of three issues and the solution of three tasks with different scales of difficulty and hence scored differently; a maximum of 20 points can be obtained, with the student having to obtain 11 points in order to obtain the minimum passing grade, i.e. 3.0).
Earn extra points for activity during class, especially for:
- discussion of additional aspects of the issue,
- efficiency of application of acquired knowledge when solving a specific task,
- early completion of the project (program with its documentation).

Programme content

The course program covers the following topics:
- basic concepts of numerical analysis (fixed-point and floating-point representations of numbers, the concept of over- and under-dimension, task conditioning, numerical correctness and stability),
- numerical implementation of calculations on polynomials and measurable functions (numerical representations of polynomials and measurable functions, Horner's algorithm, Shaw-Traub's algorithm),
- interpolation (general interpolation task, Lagrange interpolation, including the theorem on the explicitness of the solution of the Lagrange interpolation task, Newton's interpolation formula, including the concept of differential quotient, Lagrange interpolation for equidistant nodes, Hermite interpolation, including the explicitness theorem for the solution of the Hermite interpolation task and the concept of generalized differential quotients, the remainder in polynomial interpolation, measurable interpolation, trigonometric interpolation, interpolation with glued functions, including the algorithm for determining the natural and periodic glued function of the third degree),
- solving systems of linear equations (Gauss elimination, the concept of positively defined matrix, Cholesky's method, Crout's method, error estimates of matrix perturbations and free expression vector, the concept of spectral radius of matrix, Gauss-Seidel and Jacobi iterative methods, including the

theorem of comparison of spectral radii of matrices occurring in these methods),
- solving equations and systems of nonlinear equations (halving method, regula falsi, secant method, Newton-Raphson methods, Newton's general method, determination of zeros of polynomials by Bairstow method, concept of Sturm's sequence and Sturm's theorem on the number of real elements of a polynomial),
- calculation of determinants and numerical inversion of matrices,
- approximation (mean-square approximation, including the concept of Haar system, unitary approximation by polynomials, including Weierstrass theorem, unitary approximation by trigonometric polynomials).
The lecture also presents elements of interval arithmetic (arithmetic operations on intervals, the concept of interval function and interval expansion of a function of a real variable) along with its implementation on the computer (variable interval arithmetic).
As optional additional material, an overview of methods for solving other numerical problems is presented in lectures on:
- numerical differentiation (Romberg's method),
- numerical integration (Newton-Cotes quadratures, complex Newton-Cotes quadratures, Gauss quadratures, complex Gauss quadratures),
- solving the initial problem for ordinary differential equations (direct application of Taylor's formula, Adams multistep methods, Runge-Kutta methods),
- solving boundary problems for partial differential equations (differential methods).
During laboratory classes, tasks are solved on operations on intervals, application of Horner's algorithm to calculate the value of a polynomial, the remainder of dividing a polynomial by a binomial and normalized derivatives of a polynomial, application of Show-Traub's algorithm to calculate normalized derivatives of a polynomial with the demonstration of fewer operations than is the case with Horner's algorithm, construction of Lagrange and Newton interpolating polynomials based on given nodes and values of the function at the nodes, construction of Hermite's interpolating polynomial based on given nodes, tuples of nodes and values of the function and its derivatives at the nodes, determination of the number of actions in Gauss, Cholesky and Crout methods, study of convergence of Jacobi and Gauss-Seidel iterative methods, and application of Sturm's theorem to determine the number of real roots of a polynomial.

Course topics

none

Teaching methods

1. Lecture:
- multimedia presentation,
- presentation of the program content on the blackboard with proofs of selected theorems,
- PDF files for downloading (from the instructor's website) of the material presented in the lectures along with tasks for the presented content.
2. laboratory classes:
- solving tasks on the blackboard,
- execution of a program for solving a specific problem with a selected numerical method in ordinary and interval floating point arithmetic (within one laboratory group the tasks are different).

Bibliography

Basic
1. J. i M. Jankowscy, Przegląd metod i algorytmów numerycznych, Cz. 1, WNT, Warszawa.
2. A. Marciniak, D. Gregulec, J. Kaczmarek, Podstawowe procedury numeryczne w języku Turbo Pascal, Wydawnictwo NAKOM, Poznań.
Additional
1. D. Kincaid, W. Cheney, Analiza numeryczna, WNT, Warszawa.
2. Z. Fortuna, B. Macukow, J. Wąsowski, Metody numeryczne, WNT, Warszawa.
3. J. Stoer, R. Bulirsh, Wstęp do analizy numerycznej, PWN, Warszawa.
4. A. Ralston, Wstęp do analizy numerycznej, PWN, Warszawa.

Breakdown of average student's workload

	Hours	ECTS
Total workload	125	5,00
Classes requiring direct contact with the teacher	50	2,00
Student's own work (literature studies, preparation for laboratory classes/ tutorials, preparation for tests/exam, project preparation)	75	3,00